A Wavelet Based Generalized Unsharp Making Algorithm

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Abstract— In this paper we propose an improved unsharp masking algorithm. Contrast enhancement and image sharpness is required in many applications. Unsharp masking is a classical tool for sharpening an image. Unsharp masking algorithm is used for the exploratory data model as a unified framework. Proposed algorithms have three issues: 1) Contrast is increased and image is sharp by means of individual treatment of the residual and the model component. 2) Halo effect is reduced by means of wavelet based denoising methods. 3) Out-of-range problem is solved by means of log-ratio and tangent operation. Experimental result shows that the our proposed algorithm provides the better result as compared with the previous one the contrast of the image is enhanced and sharpness of the image is increased. In the proposed method the user can adjust the two parameters by controlling the contrast and sharpness to produce the better result.

Keywords—Generalized linear system, image enhancement, Unsharp masking, wavelet Denoising

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1 INTRODUCTION

Enhancing the contrast and sharpness of the images has many practical applications. Continuous research has been carried out to develop new algorithms. In this section detail review of the previous works are carried out .These related work include unsharp masking and its variants, retinex, histogram equalization and dehazing algorithm and generalized linear systems.

1.1 Related works

1) Sharpness and Contrast Enhancement:

The classical unsharp masking algorithm expressed in detail as the equation: $v = y + \gamma(x - y)$ where x is the input image,y is the result of a linear low-pass filter, and the gain $\gamma (\gamma > 0)$ is real scaling factor. The signal $d = x - y$ is amplified $(\gamma > 1)$ to increases the sharpness. The signal d contains 1) detail of the image, 2) noise, and 3) over-shoots and under-shoots in area of sharp edges due to the smoothing edges. Enhancement of the noise is clearly unacceptable; the enhancement of the under-shoot and over-shoot creates the unpleasant halo effect. This need the filter not sensitive to noise and does not have smooth sharp edges. These issues have been studied in much research. For example, the edge-preserving filter [2]-[4] and the cubic filter [1] have been used to replace the linear low-pass filter. The former is less sensitive to noise. The latter does not smooth sharp edges. Adaptive gain control has also been studied [5].

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To decreases the halo effect, edge preserving filter such as: weighted least-squares based filters [13] adaptive Gaussian filter [12] and bilateral filter [11], [14] are used. Novel algorithm for contrast enhancement in dehazing application has been published [15], [16].Unsharp masking and retinex type of algorithm is that result usually out of range of the image [12], [17]-[19]. A histogram-based a number of the internal scaling process and rescaling process are used in the retinex algorithm presented in [19]

2) Generalized linear system and the Log-Ratio Approach:

Marr [20] has pointed out that to develop an effective computer vision technique is consider: 1) Why the particular operation is used, 2) How the signal can be represented, 3) what implementation can be used. Myers presented a particular operation [21] usual addition and multiplication if via abstract analysis, more easily implemented and more generalized or abstract version of mathematical operation can be created for digital signal processing. Abstract analysis provides a way to create system with desirable properties. The generalized system is shown in Figure.1 is developed. The generalized addition and scalar multiplication denoted by ⊕ and ⊗ are defined as follows

$$x \oplus y = \phi^{-1}[\phi(x) + \phi(y)] \qquad (1)$$
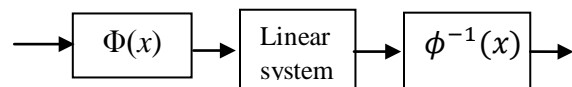
$$\alpha \otimes x = \phi^{-1}[\alpha\phi(x)] \qquad (2)$$

Fig.1   Block diagram of a generalized linear system, where φ(x) is usually a non linear function

Where x and y are the signal samples α usually a real scalar, and is a non linear function. In [17] log ratio is proposed systematically tackle out of range problem in the image

restoration. The generalized linear system point provides the log ratio point of view, where the operation are defined by using (1) and (2). Property of the log ratio is that of gray scale image set $I\epsilon(0,1)$ is closed under the new operation.

1.2 Issue Addressed, Motivation and Contributions:

In this section issues related to the contrast and sharpness enhancement is given in detail. 1) Contrast and sharpness enhancement are two similar tasks. 2) The main goal of the Unsharp is to increase the sharpness of the image and remove the halo effect. 3) While improving the contrast of the image the minute details are improved and the noise well. Contrast and sharpness enhancement have a rescaling process. It is performed carefully to provide the best result. The exploratory data model in 1)-3) and issue 4) using the log-ratio operation and a new generalized linear system is presented in this paper. This proposed work is partly motivated by the classic work in unsharp masking [1], an excellent approach of the halo effect [12], [19]. In [17] log ratio operation was defined. Motivated by the LIP model [24], we study the properties of the linear system.

2 EXPLORATORY DATA ANALYSIS MODEL FOR IMAGE ENHANCEMENT

2.1 Image model and generalized unsharp masking

In exploratory data analysis is to decompose a signal into two parts. In one part is of particular model and other part is of residual model. In tukey's own words the data model is: "data=fit PLUS residuals". ([28] Pp.208). The output of the filtering process can be denoted by the $y = f(x)$. It can be regarded as the part of the image that can be fit in the model. Thus we can show an image using generalized operation as follow

$$x = y \oplus d \tag{3}$$

Where d is called as the detail signal (the residual). The detail signal is defined as $d = x \ominus y$, $\ominus$ is the generalized operation. It provides the unified framework to study Unsharp masking algorithms. A general form of the unsharp masking is given as

$$\upsilon = h(y) \oplus g(d) \tag{4}$$

Where υ is the output of the algorithm and both $h(y)$ and $d(y)$ could be linear or non linear functions. Model explicitly states that the image sharpness is the model residual. It forces the algorithm developer to carefully select an appropriate model and avoid model such as linear filters. This model permits the incorporation of the contrast enhancement by means of suitable processing function $h(y)$ as adaptive equalization function. The generalized algorithm can enhance the overall contrast and sharpness of the image.

2.2 Outline of the Proposed Algorithm

Figure 2 shows the proposed algorithm based upon the previous model and generalized the classical Unsharp masking algorithm by addressing issues started in Section I-B. The IMF is selected due to its properties such as root signal and simplicity. Advantage of edge preserving filter is non local means filter and wavelet-based denoising filter can also be used. Rescaling process is used by the new operation defined according to the log-ratio and new generalized linear system.
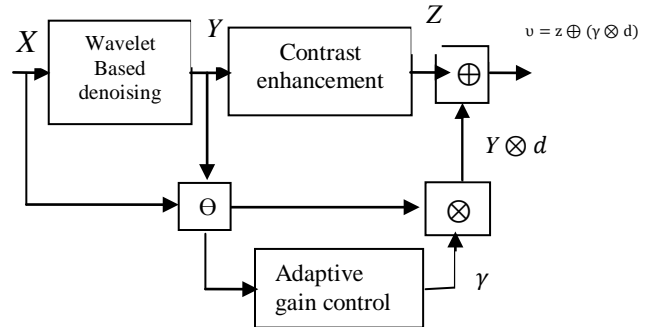

Fig. 2 Block diagram of the proposed unsharp generalized masking algorithm

3 LOG-RATIO, GENERALIZED LINEAR SYSTEMS AND BREGMAN DIVERGENCE

This Section deals with new operation using generalized linear system approach. To provide a simple presentation we use (1) and (2). These operations are defined from the vector space point of view which is similar to the development of the LIP model [26]. We provide the connection between the log-ratio, generalized systems and the Bregman divergence. As a result we show novel interpretation of two existing generalized linear systems, but also develop a new system.

3.1 Definitions and properties of log-Ratio operations

1) Non linear function:
In non linear function pixel of the gray scale of an image $x \in (0, 1)$ is considered. For an – bit image, first we add a very small positive constant to the pixel gray value then scale it by 2^{-N} such that it is in the range$(0, 1)$. The non linear function can be defined as

$$\phi(x) = \frac{\log(1-x)}{x} \tag{5}$$

To simplify the notation, we define the ratio of the negative image to the original image as follows:

$$X = \psi(x) = \frac{1-x}{x} \tag{6}$$

2) Addition and scalar Multiplication:
Using (1), the addition of two gray scales x_1 and x_2 is defined as

$$x_1 \oplus x_2 = \frac{1}{1+\psi(x_1)\psi(x_2)}$$

$$= \frac{1}{1+X_1 X_2} \tag{7}$$

Where $X_1 = \psi(x_1)$ and $X_2 = \psi(x_2)$. The multiplication of gray scale x by a real scalar $\alpha(-\infty < \alpha < \infty$ is defined by using (2) as follows:

$$\alpha \otimes x = \frac{1}{1+X^\alpha} \tag{8}$$

This operation is called as scalar multiplication which is a derived from a vector space point of view [29]. We can define a non-zero gray scale, denoted as follows:

$$e \oplus x = x \tag{9}$$

It is easy to show that $e = 1/2$. We can regard the interval $(0, (1/2))$ and $((1/2), 1)$ as the new definitions of negative and positive numbers. Absolute value is denoted as $|x|_0$ can be defined in the similar way as the absolute value of the real number as follows.

$$|x|_0 = \begin{cases} x, & \frac{1}{2} \le x < 1 \\ 1-x, & 0 < x < \frac{1}{2} \end{cases} \tag{10}$$

3) Negative Image and Subtraction Operation:
A natural extension is to describe the negative of the gray scale value. Although this can be defined by (8) and (9) the negative value of the gray scale x, denoted by x', is obtained by solving

$$x \oplus x' = \frac{1}{2} \tag{11}$$

The result is $x' = 1 - x$ which is varying with the classical definition of the negative image. This definition is also varying with the scalar multiplication in that $(-1) \otimes x = 1 - x$. The notation of the classical notation is negative which is given as: $\theta x = (-1) \otimes x$.

We can also define the subtraction operation using the addition operation in (8) as follows:

$$x_1 \theta x_2 = x_1 \oplus (\theta x_2)$$

$$= \frac{1}{\psi(x_1)\psi(\theta x_2) + 1}$$

$$= \frac{1}{X_1 X_2^{-1} + 1} \tag{12}$$

where $\psi(\theta x_2) = 1/\psi(x_2) = X_2^{-1}$ using the definition of gray scale, we also have a clear understanding of the scalar multiplication for $\alpha < 0$.

$$y = \alpha \otimes x$$

$$= (-1) \otimes (|\alpha| \otimes x)$$

$$= 1 - |\alpha| \otimes x \tag{13}$$

Here we used $\alpha = (-1) \times |\alpha|$ and the distribution law for two real scalars α and β

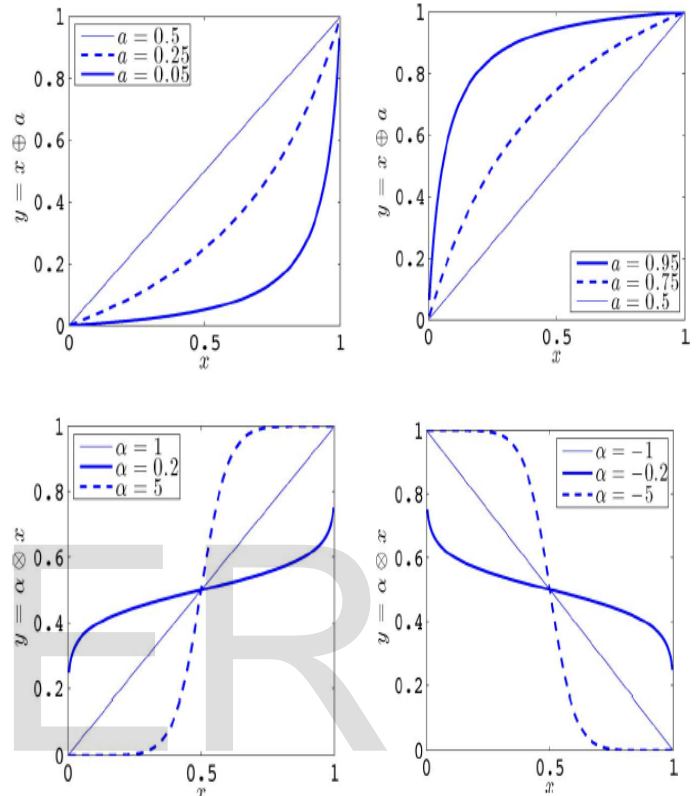$$(\alpha \times \beta) \otimes x = \alpha \otimes (\beta \otimes x) \tag{14}$$

Fig.3 Effects of the log ratio addition $y = x \oplus \alpha$ (top row) and scalar multiplication operation $y = \alpha \otimes x$ (bottom row)

3.2 Log-ratio, Generalized linear system and the Bregman divergence

We study the connection between the log-ratio and the Bregman divergence. This connection not only provides geometrical interpretation and new insight of the log-ratio, but also suggests a new way to develop generalized linear systems.

1) Log-Ratio and The Bregman Divergence:
The classical weighted average can be regarded as the solution of the following optimization problem:

$$u_{WA} = arg \ \min_u \sum_{n=1}^N \alpha_n (x_n - u)^2 \tag{15}$$

What is the corresponding optimization problem that leads to the generalized weighted average stated in (19)?

To study this problem, we need to recall some result in the Bregman divergence [30], [31]. The Bregman divergence of two vectors x and y, denoted by $D_F(x \mid y)$, is defined as

follows:

$$D_F(x,y) = F(x) - F(y) - (x-y)^T \nabla F(y) \qquad (16)$$

Where $F: X \to R$ convex and differentiable function is defined over an open convex domain X and $\nabla F(y)$ is the gradient of F evaluated at the point y. Centroid of a set of vector denoted $\{x_n\}_{n=1}: N$ in terms of minimizing the sum of the Bregman divergence is studied in a recent paper [31]. The weighted left-sided centriod is given by

$$c_L = \arg\min_{c \in x} \sum_{n=1}^N \alpha_n D_F(c \| x_n)$$

$$= \nabla F^{-1}(\sum_{n=1}^N \alpha_n \nabla F(x_n)) \qquad (17)$$

Comparing (19) and (22), we can see that when x_n a scalar is, the generalized weighted average of the log-ratio is a special case of the weighted left-sided centriod with $\phi(x) = \nabla F(x)$. It easy to show that

$$F(x) = \int \phi(x) dx \qquad (18)$$

$$= -x \log(x) - (1-x) \log(1-x) \qquad (19)$$

where the constant of the indefinite integral is omitted. $F(x)$ is called the bit entropy and the corresponding Bregman divergence is defined as

$$D_F(x \| y) = -x \log\frac{x}{y} - (1-x) \log\frac{1-x}{1-y} \qquad (20)$$

Which is called the logistic loss therefore, the log-ratio has an intrinsic connection with the Bregman divergence through the generalized weighted average. This connection reveals a geometrical property of the log-ratio which uses a particular Bregman divergence to measure the generalized distance between two points. It is compared with the weighted average which uses the Euclidean distance. Loss function of log-ratio uses the logistic loss function; the classical weighted average uses the square loss function.

2) Generalized Linear system and the Bregman Divergence:

The connection between the Bregman divergence with other well establish generalized linear system such as the MHS with $\phi(x) = \log(x)$ where $x \epsilon (0, \infty)$ and the LIP model [26] with $\phi(x) = -\log(1-x)$ where $x \epsilon (-\infty, 1)$. The corresponding Bregman divergences are the Kullback-Leibler (KL) divergence for the MHS [31]

$$D_F(x,y) = x \log\frac{x}{y} - (x-y) \qquad (21)$$

and the LIP

$$D_F(x,y) = (1-x) \log\frac{1-x}{1-y} - [(1-x) - (1-y)] \qquad (22)$$

LIP model demonstrate the information-theoretic interpretation. The relationship between the KL divergence

and the LIP model reveals a novel into its geometrical property.

3) A New Generalized Linear System:

In Bregman divergence corresponding generalized weighted average can be defined as $\phi(x) = \nabla F(x)$. For example the log-ratio, MHS and lip can be developed from the Bregman divergences. Bregman divergence measures the distance of two signal samples. The measure is related to the geometrical properties of two signal space. Generalized linear system for solving the out-of-0range problem can be developed by the following Bregman divergence (called "Hellinger-like" divergence in Table I on [31])

$$Df(x,y) = \frac{1-xy}{\sqrt{1-y^2}} - \sqrt{1-x^2} \qquad (23)$$

which is generated by the convex function $F(x) = -\sqrt{1-x^2}$ whose domain is (-1, 1). The nonlinear function $\phi(x)$ for the corresponding generalized linear system is as follows

$$\phi(x) = \frac{d(F(x))}{dx} = \frac{x}{\sqrt{1-x^2}} \qquad (24)$$

In this paper the generalized linear system is called the tangent system and the new addition and scalar multiplication operation are called tangent operations. In image processing application, first linearly map pixel value from the interval $[0, 2^N]$ to a new interval (-1, 1).

Then the image is processed by using the tangent operation. The result is then mapped back to the interval $(0, 2^N)$ through inversing mapping. We can verify the signal with the signal set $I \epsilon (-1, 1)$ is closed under the tangent operations. The tangent system can be used as an alternative to the log-ratio to solve the out-of-range problem. The application and the properties of the tangent operation can be studied in a similar way as those presented Section III-A. The negative image and the subtraction operation, and study the order relation for the tangent operations. As shown in Figure.5 the result of adding a constant to an N-bit image (N=8) using the tangent addition. In simulation, we use a simple function $q(x) = \frac{2(x+1)}{2^N+1} - 1$ to map the image from $[0, 2^N]$ to (-1, 1). We can see that the effect is similar to the log-ratio addition.

4 PROPOSED ALGORITHM

4.1 Dealing with color images

First the color image is converted from the RGB color space to the HIS or the LAB color space. The chrominance components such as the H and S components are not processed. After the luminance component is processed the inverse conversion is performed. Enhanced color image in RGB is obtained. Rationale processing is carried out in luminance component to avoid a potential problem of altering the white balance of the image when the RGB components are processed individually two iteration $H(y_k, y_{k+1}) = (1/N) \|y_k - y_{k+1}\|^{\frac{2}{2}}$ where N is the number of pixels in the image Result using two setting of the wavelet based denoising

and using the "cameraman" image are shown.

4.2 Enhancement of the detail Signal

1). The root Signal and the Detail Signal :

Let us denote the median filtering operation as a function $y = f(x)$ which maps the input x to the output y. An IMF operation can be denoted as: $y_{k+1} = f(y_k)$ where $k = 0, 1, 2, \ldots$ is the iterantion index and $y_0 = x$. The signal y_n is usually called the root signal of the filtering process if $y_{n+1} = y_n$. It is convenient to define a root signal y_n as follows:

$$n = \min k \, , subject \ to \ H(y_k, y_{k+1}) < \delta \qquad (25)$$

Where $H(y_k, y_{k+1})$ is a sultable measur of the difference between the two images. δ is a user defined threshold. For natural image, mean square difference, defined as $H(y_k, y_{k+1}) = \left(\frac{1}{N}\right) \| y_k - y_{k+1} \| = (1/N) \| (y_k - y_{k+1} \|_2^2$ (N is the number of the pixels), is a monotonic decreasing function of K. An example is shown is figure below. It is clear that the definition of the thershold is depends upon the threshold. It is possible to set a large value of the δ such that y_1 is the root signal. After five iteration ($k \geq 5$) the difference $H(y_k, y_{k+1})$ changes occurs slightly. We can regard y_4 or y_5 the root signal. Of course, the number of the iterations, the size and the shape of the filter mask has certain impact on the root signal. The original signal in shown in Figure. 7. Which is the 100th row of the "cameraman" image? The root signal y is produced by an IMF filter with a (3×3) mask and the three iteration. The signal s is produced by a linear low-pass filter with a uniform mask of (5×5). The gain of the both algorithm is three. On comparing the enhanced signal we can see clearly that while the result for the classical unsharp masking algorithm suffers from the out of range problem and halo effect (under-shoot and over-shoot), the result of the proposed algorithm is free of such problem.

2)   Adaptive Gain Control:

In Figure.3 to enhance the detail the gain must be greater than one. Using a universal gain for the whole image does not lead to good results, because to enhance the small detail a relatively large gain is needed. A large gain can lead to the saturation of the detailed signal whose values are larger than the threshold. Saturation is undesirable because different amplitude of the detail signal is mapped to the same amplitude of either 1 or 0. This leads to loss of information. The gain must be adaptively controlled.

We describe the gain control algorithm for using with the log-ratio operation. To control the gain, linear mapping of the detail signal d to a new signal c.

$$c = 2d - 1 \qquad (26)$$

Such that the dynamic range of c is (-1,1). A simple idea is to set the gain as a function of the signal c and to gradually decrease the gain from its maximum value γ_{MAX} when $c \mid < T$ to ots minimum value γ_{MIN} when $\mid c \to 1$. We propose the following adaptive gain function

$$\gamma(c) = \alpha + \beta \exp(-|c^\eta|) \qquad (27)$$

where η is a parameter that controlsthe rate of decreasing. The parameter α and β are obtained by solving the equation: $\gamma(0) = \gamma_{MAX}$ and $\gamma(1) = \gamma_{MIN}$. For a fixed η, we can easily determine the parameter as follows:

$$\beta = (\gamma_{MAX} - \gamma_{MIN})/(1 - e^{-1}) \qquad (28)$$
and

$$\alpha = \gamma_{MAX} - \beta \qquad (29)$$

Both γ_{MAX} and γ_{MIN} could be chosen based upon each individual image processing task. It is reasonable to set $\gamma_{MIN} = 1$. This setting follows the intuition that when the amplitude of the detailed signal is large enough, it does not need any further amplification. For example we can see that

$$\lim_{|d|_0} \gamma \otimes d = \lim_{d_0 \to 1} \frac{1}{1+\left(\frac{1-d}{d}\right)\gamma} = 1 \qquad (30)$$

Scalar multiplication has little effect. We now study the effect of η and γ_{MAx} by setting $\gamma_{MIN} = 1$.

4.3 Contrast Enhancement of the Root Signal

For contrast enhancement, we use adaptive histogram equalization implement by Matlab function in the Image processing Toolbox. The function called "adapthisteq", has a parameter controlling the contrast. This parameter is determined by user through experiment to obtain the most visually pleasing result. In simulation, we use default values for other parameters if the function

5 WAVELET DENOISING

The wavelet transform has been a powerful and widely used tool in image denoising because of its energy compaction and multi resolution properties. Denoising an image corrupted with additive white Gaussian noise was initially proposed by thresholding the wavelet coefficients. Subsequently, various decomposition strategies and thresholding schemes have been proposed. However, most of these use classical orthogonal wavelets which are independent of the image and noise characteristics and focus on finding the best threshold. Unlike the Fourier transform with its complex exponential basis, the wavelet transforms do not have a unique basis. Noting this point several attempts at designing matched wavelets have been made with the goal of match varying from match to a signal and energy compaction to maximizing the signal energy in the scaling sub-space.

The most important way of distinguishing information from noise in the wavelet domain consists of thresholding the wavelet coefficients. Mainly hard and soft thresholding techniques are performed. Thresholding is the simplest method of image denoising .In this from a gray scale image,

thresholding can be used to create binary image. Thresholding is used to segment an image by setting all pixels whose intensity values are above a threshold to a foreground value and all the remaining pixels to a background value. Thresholding is mainly divided into two categories

Hard threshold is a "keep or kill" procedure and is more intuitively appealing. The transfer function of the hard thresholding is shown in the figure. Hard thresholding may seem to be natural. Sometimes pure noise coefficients may pass the hard threshold.

Soft threshold shrinks coefficients above the threshold in absolute value. The false structures in hard thresholding can be overcome by soft thresholding. Now a days, wavelet based denoising methods have received a greater attention

6 EXPERIMENTAL ANALYSIS

Here we considered wavelet based Denoising using hard and soft thresholding approaches as stated in [22]. We have tested our experimented five different images samples compared against different parameters and the output results of the proposed methods are shown below

Fig. 4 Original Image

Fig. 5 Detailed Coefficients

Fig. 6 Proposed method with HT thresholding

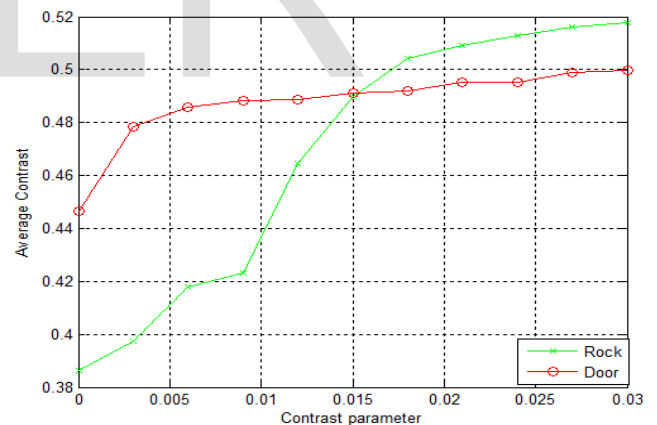Fig. 7 Proposed method with ST Thresholding

Fig. 8 Performance analysis contrast parameter Vs average contrast for different images

6 CONCLUSION

In this paper an improved approach for classical unsharp masking algorithm is proposed by introducing wavelet based thresholding. From the above obtained results we can conclude that the proposed method out performs for different images under different block size and average contrast parameter. This method can significantly increase the sharpness and contrast ratios appropriately

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APPENDIX

TABLE IV

Key components of some generalized linear system motivated by the bregman divergence. The domain of the lip model is $(-\infty, M)$. In this table, it is normalized by M to simplify notations

	Domain	$D_F(x, y)$	$\phi(x)$	$x \oplus y$	$\alpha \otimes x, (\alpha \in R)$
Log-ratio	(0, 1)	$-x\log\dfrac{x}{y} - (1-x)\log\dfrac{1-x}{1-y}$	$\log\dfrac{1-x}{x}$	$\dfrac{1}{1+\frac{1-x}{x}\frac{1-y}{y}}$	$\dfrac{1}{1+\left(\frac{1-x}{x}\right)^\alpha}$
LIP	$(-\infty, 1)$	$(1-x)\log\dfrac{1-x}{1-y} - [(1-x)-(1-y)]$	$-\log(1-x)$	$x+y-xy$	$1-(1-x)^2$
MHS	$(0,\infty)$	$x\log\dfrac{x}{y} - (x-y)$	$-\log(x)$	xy	x^α
Tangent	$(-1, 1)$	$\dfrac{1-xy}{\sqrt{1-y^2}} - \sqrt{1-x^2}$	$\dfrac{x}{\sqrt{1-x^2}}$	$\dfrac{\phi(x)+\phi(y)}{\sqrt{1+(\phi(x)+\phi(y))^2}}$	$\dfrac{\alpha\phi(x)}{\sqrt{1+(\alpha\phi(x))^2}}$